Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

246. Proposed by ALBERT A. BENNETT, Princeton University.

Prove that

$$\frac{1}{\sqrt{b}}\left[\left(\frac{a+\sqrt{b}}{2}\right)^n - \left(\frac{a-\sqrt{b}}{2}\right)^n\right]$$

is an integer for every positive integral value of n, whenever a is an odd integer, positive or negative, and $b \equiv 1$ (mod. 4).

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

Taking out the factor

$$\frac{a+\sqrt{b}}{2} - \frac{a-\sqrt{b}}{2} = \sqrt{b},$$

we have

$$\frac{1}{\sqrt{b}}\left[\left(\frac{a+\sqrt{b}}{2}\right)^n - \left(\frac{a-\sqrt{b}}{2}\right)^n\right] = \sum_{s=0}^{n-1}\left(\frac{a+\sqrt{b}}{2}\right)^{n-s-1}\left(\frac{a-\sqrt{b}}{2}\right)^s.$$

Let

$$\frac{a+\sqrt{b}}{2} = x, \qquad \frac{a-\sqrt{b}}{2} = y.$$

Then this expression is

$$\text{(I)} \qquad (x^{n-1} + y^{n-1}) + xy(x^{n-3} + y^{n-3}) + x^2y^2(x^{n-5} + y^{n-5}) + \cdots + L;$$

where $L = x^{(n-1)/2}y^{(n-1)/2}$ if n is odd; and $L \equiv x^{(n-2)/2}y^{(n-2)/2}(x+y)$ if n is even.

Let $a = 2k + 1$, $b = 4l + 1$, k and l being integers. Then

$$xy = \frac{a^2 - b}{4} = k^2 + k - l,$$

an integer, so that any positive integral power of xy is an integer.

Also $x + y = a$ and $x^2 + y^2 = (x + y)^2 - 2xy = a^2 - 2(k^2 + k - l)$ are integers. Further,

$$x^p + y^p = (x + y)^p - \Sigma \frac{p!}{s!(p-s)!} x^s y^s (x^{p-2s} + y^{p-2s});$$

so that if $x^p + y^p$ is integral for a positive integral value of p, it is integral for a value of p greater by 2; but it is integral for $p = 1$ and for $p = 2$, and so is integral for any positive integral value of p.

Thus each term of (I) is integral, and I is an integer.

Also solved by ELBERT H. CLARKE.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence.

DISCUSSIONS.

I. RELATING TO INFINITESIMAL METHODS IN GEOMETRY.

BY E. B. WILSON, Massachusetts Institute of Technology.

The old method of using infinitesimals to find by geometric considerations various properties of figures was given a prominent place by W. E. Byerly in his *Differential Calculus,* but has been almost wholly discarded by recent writers. It is a question worthy of discussion whether the present neglect of the method is entirely wise. An example or two may be given.